

MYRON MATHISSON: WHAT LITTLE WE KNOW OF HIS LIFE*

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Myron Mathisson (1897–1940) was a Polish Jew known for his work on the equations of motion of bodies in general relativity and for developing a new method to analyze the properties of fundamental solutions of linear hyperbolic differential equations. In particular, he derived the equations for a spinning body moving in a gravitational field and proved, in a special case, the Hadamard conjecture on the class of equations that satisfy the Huygens principle. His work still exerts influence on current research. Drawing on various archival and secondary sources, in particular his correspondence with Einstein, we outline Mathisson's biography and scientific career.

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Early years

Myron Mathisson was born in Warsaw on 14 December 1897¹. Very little is known about his family background. His parents, Hirsh and Khana Mathisson, had moved from Riga to Warsaw (IG). We do not know the

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¹ Unless stated otherwise, biographical information about Mathisson's early life is taken from a letter to Einstein, dated 23 February 1930 (AEA 18-004). See the Appendix for a synopsis of the available sources for biographical information about Mathisson and a resolution of our abbreviations such as (AEA) and (UW). On his high school diploma (UW), issued on 1 May 1915, the date of his birth is given, in the old-style (Julian) calendar, as 2 December 1897. This corresponds to 14 December 1897 in the Gregorian calendar. In the twentieth century, the difference between those calendars was 13 days; this may have led to the fact that some sources, including AEA 18-004, give December 15 as the relevant date.



Fig. 1. Myron Mathisson (1897–1940)

profession or business of Myron's father, but the family must have been relatively well-off, as may be inferred from their address in a well-to-do area of Warsaw (Polna 70, now Noakowskiego 16), as well as from the excellent education received by Mathisson. He attended, from 1906 to 1915, a Russian philological gymnasium named after *The Great Prince Aleksy Nikolayevich, The Successor to the Throne*, one of the best secondary schools in Warsaw at the time. His high school diploma contains grades in Russian, Latin, Greek, German and French (Polish was then not allowed in schools), but also in mathematics and physics; all his grades were excellent, and he graduated with a gold medal.

We do not know what language was spoken in the Mathisson household; Myron knew perfectly well Hebrew, Russian, and Polish; he never mentioned Yiddish.

1915–31 studies and Ph.D.

On 3 December 1915, Mathisson started studies at the Department of Civil Engineering of Politechnika Warszawska, which at that time was already a Polish language school. (Recall that, as a result of the war, the Russians were forced, by the German army, to leave Warsaw.) He explained later in a letter to Albert Einstein that his decision to study at this school had been influenced by the fact that excellent French mathematicians and

physicists of the times of Augustin Fresnel had studied at the École Polytechnique. One can guess that he soon realized that the Warsaw Polytechnic was not quite like the French one: in 1917 he started to follow the activity of the physics laboratory at Warsaw University.

On 11 June 1917, Mathisson received a severe reprimand from the Senate of the Warsaw Polytechnic for “having signed an indecent and improper polemic with the Rector and Senate of Politechnika Warszawska” (PW). It is unknown, however, what the “polemic” had been about. After passing the First Diploma examination in 1918, he quit the Polytechnic and, in 1919, was drafted into the Polish army, then fighting the Soviets.

After the war, he tried, unsuccessfully, to resume studies at the Warsaw Polytechnic. In the Archives of that school, there is a very polite letter by Mathisson, dated 6 October 1920, addressed to the Rector, asking for readmission. The official who dealt with the case noted on the margin that Mathisson had left the Polytechnic after having received a reprimand and submitted his case to the Senate; this led to a rejection (PW).

In the end the refusal turned out good for science because, as a result, in the fall of 1920, Mathisson entered Warsaw University to study physics. However, the death of his father, some time in the early 1920s, resulted in a deterioration of the financial situation of the family and forced him to interrupt his studies.

In 1925 Mathisson wrote his first scientific paper *On the motion of a rotating body in a gravitational field*. Czesław Białobrzewski, the only professor of theoretical physics at Warsaw University at the time, was ready to accept it as a Ph.D. thesis, but Mathisson became dissatisfied with his own work, left the tutelage of Białobrzewski without even saying a word of good bye, as he later admitted to Einstein, and went to Palestine. It is unclear how long Mathisson stayed in Palestine, but a severe tropical illness, probably malaria, forced him to return to Poland.

After having returned to Warsaw, he continued to work, in solitude, on the problem of motion in the general theory of relativity. His small income was derived from giving Hebrew lessons and performing some auxiliary tasks for the construction engineers.

On 18 December 1929, Mathisson started his correspondence with Albert Einstein² with a long first letter (see Fig. 2). On reading it, one is

² The Albert Einstein Archives (AEA) contains 19 letters by Mathisson to Einstein, and 11 letters by Einstein to Mathisson. In addition, the archives contain an unpublished manuscript by Mathisson (AEA 18-030) and more than 30 letters that mention Mathisson. All of Mathisson’s correspondence is hand-written, while all of Einstein’s extant letters in the archives are preserved as carbon copies of typewritten letters. It is clear, both from gaps in the sequence of the correspondence and from reference to missing letters, that not all of Einstein’s letters have been preserved. Presumably, it is the handwritten letters by Einstein that we are missing.

impressed by the quality of his French and the audacity of his criticism of Einstein's paper: Mathisson writes that, in [1], Einstein neglects radiation and deviations from spherical symmetry, these approximations being due to the "mathematical insufficiency of your method".

Monsieur,

Voire Mémoire, publié dans les SITZUNGSBERICHTE DER PREUSS. AKADEMIE (séance du 8. Décembre 1927), intitulé ALLGEMEINE RELATIVITÄTSTHEORIE UND BEWEGUNGSGESETZ, s'occupe d'un problème, dont j'ai établi, il y a deux ans et demi, une solution plus complète.

Fig. 2. The beginning of Mathisson's first letter to Einstein, 18 December 1929 (AEA 18-001).

At the time of writing this letter, the 32-year-old Mathisson did not have a Ph.D. nor any publications to his reputation. Yet, in this 11-page letter Mathisson described, in general terms, without equations, his views on the problem of motion in general relativity, boldly criticizing Einstein's own approach. He also gave an indication of his difficult financial situation. He announced that he would send Einstein a manuscript detailing his ideas, but said that, should this manuscript be published, he would have to ask for some financial remuneration for it.

Einstein's reply to Mathisson's first letter is unknown, but from Mathisson's second letter, dated 14 February 1930 (AEA 18-002), we can infer that Einstein had replied obligingly by inviting Mathisson to come to Berlin to collaborate with him in some form. Einstein at the time was, of course, a world-famous man. Scientifically, he was deeply involved in investigating the implications of his teleparallel approach toward unified field theory, and he was, in fact, just looking for another mathematically trained collaborator [2]. His long-time assistant Jakob Grommer had just left or was just about to leave to take up a position in Minsk. Cornelius (Kornél) Lánosz, who had spent a year with Einstein in Berlin to work with him on the teleparallel approach, was leaving as well. In early 1930, Walther Mayer came to Berlin and became Einstein's mathematical collaborator for the next few years. We should also like to mention that Einstein himself had written similarly audacious letters to scientific authorities such as Paul Drude, when he

had still been a young man, unknown to the scientific world. Mathisson declined the invitation on the ground that he did not yet feel prepared for such a collaboration. But he did send a manuscript on the problem of motion.

An exchange of several letters followed; in his response of 20 February 1930 (AEA 18-003), Einstein wrote that a superficial reading of Mathisson's paper had already convinced him "that you have an extraordinary formal talent and that you are made for scientific work". He suggested that Mathisson's manuscript be submitted as a Ph.D. thesis. He offered to try to obtain a fellowship that would enable Mathisson to come to Berlin and work with him. To this end, he asked Mathisson for further information about his life, academic training, and profession. In response, Mathisson sent a third letter, dated 23 February 1930 (AEA 18-004), in which he gave a curriculum vitae of sorts, another outline of his ideas and comments on his personal and intellectual situation. This letter of 14 pages is one of the most important sources for our knowledge about Mathisson's early life and work.

In response to Mathisson's third letter, Einstein forwarded Mathisson's manuscript to Białobrzęski and suggested that Mathisson be awarded a Ph.D. degree on the basis of his results on the problem of motion, apologizing for Mathisson's earlier "odd behavior". He also offered to cover the expenses connected with presenting the thesis (AEA 18-005, 18-006). Despite his enthusiasm for Mathisson's work, Einstein also asked whether "it might be possible to make do without Schouten's terrible notation? This would be such a relief for the reading mankind". The notation in question concerned integrals over n -dimensional manifolds. Mathisson did not follow Einstein's suggestion and, in later letters (AEA 18-027 and 18-034), explained how this precise notation helped him to obtain new results.

On 31 October 1930 M. Mathison³ obtained the Ph.D. degree from Warsaw University on the basis of a thesis on *General relativity theory and the dynamics of the electron* (in Polish) (UW).

The text of the thesis has not been preserved, but its results are presented in Mathisson's first three publications. These appeared in 1931 in *Zeitschrift für Physik* [M1-3]⁴ with Einstein's help. A week after obtaining his Ph.D., Mathisson wrote to Einstein informing him of his graduation as well as about the Rockefeller Foundation's negative decision (see below; AEA 18-029). He expressed his hope to obtain the scholarship if he succeeded in publishing his results. He indicated that he would like to come to Berlin to see Einstein

³ This spelling was in all legal documents, such as his passport, high school certificate, and student identity card; in his letters and publications, he always spelled his name with a double s, a logical thing to do, considering the Latin form (Matthaeus) of the name Matthew.

⁴ In this style we refer to the list of *Scientific publications of Myron Mathisson* included in the last section of this paper.

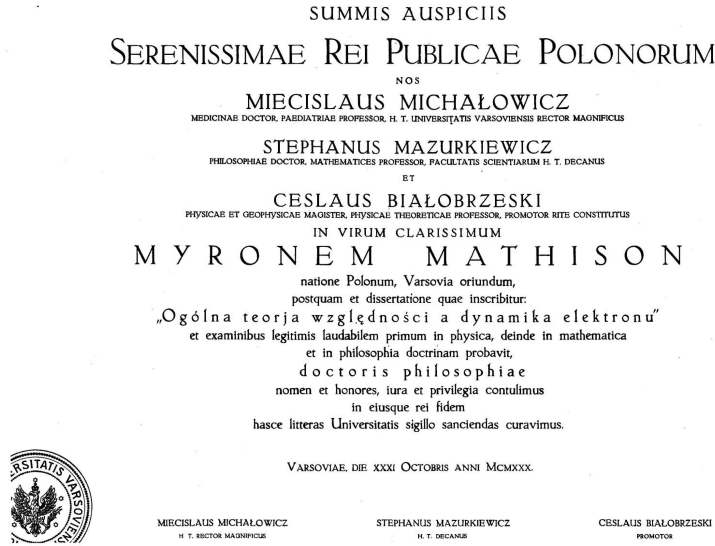


Fig. 3. Myron Mathisson's doctoral diploma (UW). Note the spelling of his name.

but also because that would bring him “nearer to his final destination — Palestine”. He also included a manuscript, a “small physical sketch” as he called it, entitled “Relativistic Formulation of Quantum Phenomena” (AEA 18-030).

In his response (AEA 18-031), Einstein rejected Mathisson's ideas on the quantum problems on the ground that his approach would be equivalent to the old, non-relativistic quantum theory based on the Bohr–Sommerfeld quantization rule $\int pdq = nh$. In response to a follow-up letter by Mathisson, in which he indicated his intention to publish his relativistic results of the last three years (AEA 18-032), Einstein sent Mathisson two letters of recommendation “on the basis of which your relativistic works, that are certainly very valuable, will be accepted with certainty” (AEA 18-033).

The first publications

Mathisson's first three papers indeed appeared in *Zeitschrift für Physik*. [M1] was received by that journal on 8 December 1930, [M2] on 23 December 1930, and [M3] on 19 March 1931.

Those papers contain an implicit polemic with Einstein and his approach to the problem of motion. The first paper contains an essential generalization of Einstein's linearization of the field equations [3]. Mathisson allows the background metric to be curved. In the second, Mathisson shows that nonlinearity of the field equations is not essential for obtaining from

them the equations of motion. He uses extensively the geometry of the null cone introduced earlier by Hermann Minkowski [4]. In flat spacetime, given a time-like world-line $z^\mu(s)$,⁵ he introduces two functions of $x = (x^\mu)$: a null coordinate s and a co-moving radial distance r , such that the vector $l^\mu(x) = x^\mu - z^\mu(s(x))$ is null and oriented towards the future and $r = \dot{z}_\mu l^\mu$. If the electromagnetic potential is assumed to have the form $A^\mu = e^\mu(s)/r$ so that it satisfies $\square A^\mu = 0$, then the Lorentz condition $A^\mu_{;\mu} = 0$ implies $e^\mu(s) = e(s)\dot{z}^\mu(s)$ and the conservation of charge, $\dot{e} = 0$. Therefore, the Liénard–Wiechert potential $A^\mu = e\dot{z}^\mu/r$ satisfies Maxwell’s equations for an arbitrary motion of the charge. After describing this, Mathisson [M2] then considers an analogous problem for a weak gravitational field $h_{\mu\nu}$. He introduces, following Einstein, the tensor $\psi^\nu_\mu = h^\nu_\mu - \frac{1}{2}\delta^\nu_\mu h^\rho_\rho$ so that the linearized field equations in empty space are equivalent to $\square\psi^\nu_\mu = 0$ and $\psi^\nu_{\mu,\nu} = 0$. The tensor field $\psi^{\mu\nu} = m^{\mu\nu}(s)/r$ satisfies the wave equation and the Einstein condition (so called in [M3]) $\psi^\nu_{\mu,\nu} = 0$ implies $m^{\mu\nu} = m\dot{z}^\mu\dot{z}^\nu$, $\dot{m} = 0$ and the equation of motion $\ddot{z}^\mu = 0$. In the third paper, he mentions the possibility of using Dirac’s delta functions, a novelty for relativists at that time. He gives a new derivation of the special relativistic equation of motion of a charge, taking into account its self-interaction, resulting in the radiative reaction force $\frac{2}{3}e^2(\ddot{u}^\nu u_\nu u^\mu - \ddot{u}^\mu)$, where $u^\mu = \dot{z}^\mu$.

Early in 1931, Mathisson wrote his first paper on a new approach to the study of fundamental solutions of partial differential equations of the hyperbolic type; in mid-March, he sent a manuscript, entitled “A parametrix method for generalized wave equations in Riemannian manifolds” to Einstein. With a little delay due to his being absent from Berlin, the paper was forwarded by Einstein to Otto Blumenthal, managing editor of *Mathematische Annalen*. Einstein, who had been co-editor of the journal from 1919 to 1928, recommended the paper for publication, saying

I know the author from earlier works as a very intelligent and diligent writer who is dealing with the deepest problems of the general theory of relativity under very difficult external conditions. (AEA 18-036)

In a postcard written a day later, Blumenthal acknowledged receipt of the manuscript, but requested that Einstein make more specific comments on the manuscript itself. No paper with that title ever appeared in the *Mathematische Annalen*, and we may conjecture that it appeared only later with a similar title in 1934 in the Polish journal *Prace Matematyczno-Fizyczne* [M5]. Nevertheless, a paper by Mathisson on the same subject, but with

⁵ We use the Greek alphabet to denote spacetime indices. This is in agreement with the prevalent current notation, but Mathisson, in his first papers, used Latin indices. The index ν after a comma (respectively, semicolon) denotes the ordinary (respectively, covariant) derivative in the direction of the vector $\partial/\partial x^\nu$.

different title [M4], appeared later in the *Mathematische Annalen*. This paper, however, was only received by the journal on 21 December 1931, and its publication was probably endorsed by Jacques Hadamard. In this and in later papers on this subject, Mathisson made essential use of methods of differential geometry, tensor calculus, and the geometry of the null cone. In his letter of 25 April 1931 (AEA 18-034), he emphasized the essential role that tensor calculus and Riemannian geometry had played in his derivation of an integral formula in the paper that he was sending then to Einstein.

Efforts to obtain a Rockefeller stipend for Mathisson

In 1930, after reading Mathisson's first letters and manuscript, Einstein sent several letters to various officers of the *Notgemeinschaft der Deutschen Wissenschaft* and of the Rockefeller Foundation to inquire about the proper procedure of applying for a stipend for Mathisson. As it turned out, the rules required that the application be filed not by Einstein but by a mentor of the candidate's home country, Białobrzski in Mathisson's case⁶. More importantly, several conditions had to be satisfied. The candidate had to present an invitation by a scholar abroad whose guest he would be, he had to hold "at least" a Ph.D. and have scientific publications under his name, and he had to have an academic position in his home country to which he would be able to return after spending time abroad. Only the first condition was satisfied in Mathisson's case. So Einstein wrote another diplomatic letter to Białobrzski (AEA 18-009), asking him to submit the application on Mathisson's behalf, and, in particular, to be generous in his assurances of a future academic career for Mathisson:

It will hardly be avoidable, for the sake of a good cause, to pay tribute, albeit somewhat insincerely, to the almighty Bureaucratius, which a higher authority will readily forgive because of its unreasonableness.
(AEA 18-009)

Białobrzski complied with Einstein's wish (AEA 18-010), and Einstein, on his part, sent a letter of recommendation in support of Mathisson to the Rockefeller Foundation (AEA 18-011).

In June 1930, a Rockefeller delegate met with Mathisson, but missed Białobrzski. Informed by Białobrzski about the failed meeting, Einstein wrote another letter to the Rockefeller Foundation, confirming his support of Mathisson's application. In spite of Einstein's intervention, the Rockefeller Foundation informed Białobrzski that its Paris office would withhold

⁶ For a historical account of the role of scholarships awarded by the International Educational Board of the Rockefeller Foundation for the internationalization of mathematics in the twenties of the last century, see [5].

the application and not forward it to the Committee of the International Educational Board in the United States until Mathisson had fulfilled the necessary requirements.

During his visit in the United States, on 14 December 1930, Einstein had a chance to meet John D. Rockefeller and his wife in person (RA). The meeting was arranged on Einstein's initiative and during their conversation, Einstein tried to convince Rockefeller that his rules for awarding scholarships were too strict and often defeated the very purpose of the original intent of the scholarship program. In an account of the meeting by a mutual acquaintance, George Sylvester Viereck, who had arranged it, we read

Professor Einstein argued with him that the strict regulations laid down by his educational foundations sometimes stifled the man of genius. "Red tape," the professor exclaimed, "encases the spirit like the bands of a mummy." Rockefeller, on the other hand, pointed out the necessity for carefully guarding the funds of the foundations from diversion to unworthy ends or individuals who are not most meritorious. Standing his ground against the greatest mind in the modern world, he ably defended the system under which the various foundations were conducted. "I," Einstein said, "put faith in intuition". "I," Rockefeller replied, "put my faith in organization". Einstein pleaded for the exceptional man. Rockefeller championed the greatest good of the greatest possible number. Einstein was the aristocrat, Rockefeller the democrat. Each was sincere, each without convincing the other, persuaded him of his sincerity [6].

Mathisson, as it turned out, would become a victim of this conflict of opinions. The immediate outcome of Einstein's visit to Rockefeller was a promise that Rockefeller himself would look into any particular case that Einstein might point out to him where the rules of his foundation exclude a worthy recipient. After his return to Berlin in spring 1931, Einstein wrote to Mathisson that he wanted to make another attempt to obtain a Rockefeller scholarship for him (AEA 18-035). Now that Mathisson had obtained his Ph.D. and had also a couple of scientific publications, most of the earlier obstacles had been removed. The only condition that was still violated was the requirement that Mathisson be given assurances of a permanent academic position in Poland to return to after his stay abroad. Once again, an application for a scholarship was formally filed but again it seems to have never been forwarded from the European office in Paris to the awarding committee in the US. In June 1931, Einstein even wrote to Rockefeller himself, mentioning Mathisson as a case in point of their earlier dispute (AEA 18-049). In a polite response, Rockefeller promised to look into this but indicated that he could do so only after the summer when the committee would meet again (AEA 18-050). This was very much the end of Einstein's attempt to obtain a Rockefeller scholarship for Mathisson.

In September 1931, Mathisson wrote to Einstein (AEA 18-046), in a somewhat depressed mood, that the Rockefeller Foundation once more had decided not to forward his application, that he had saved some money and could visit Einstein in Berlin; he also mentioned that he was considering looking for a job in Russia or in Palestine, because “*in Polen ist kein Platz da für Leute meiner Nationalität*”. The visit was not realized because Einstein was then about to go to the United States. He wrote back that one should first wait what would come of his intervention with Rockefeller, and then see about this after his return to Germany (AEA 18-048). As it turned out, neither did Mathisson ever receive a scholarship from the Rockefeller foundation, nor did Einstein and Mathisson ever meet.

1932–35: habilitation, lectures in Warsaw and first visit to Paris

In 1932, Mathisson obtained a habilitation at Warsaw University that allowed him to use the title of “docent” (analog of the German *Privat-Dozent*) and give lectures (already during the year 1932–33), but did not imply a permanent position (UW).

During the years 1932–36, Mathisson gave at Warsaw University, in his capacity of docent, several courses of lectures: *Kinetic theory of gases*, *Applications of the theory of groups to quantum theory* (based on Wigner’s book), *Theory of relativity*, *Tensor calculus* (according to Schouten), *Cosmology* and *Theoretical Physics* for students of chemistry. He also conducted, together with Białobrzewski and Otton Nikodym, the main seminar on theoretical physics (UW and HU).

Mathisson’s work on partial differential equations attracted the attention of Hadamard who, in 1933, wrote to Einstein saying that his recommendation could secure a stipend for Mathisson to come to Paris (AEA 12-040). Einstein, then himself a refugee in Belgium, wrote such a letter of recommendation for Mathisson to the mathematician Paul Montel in Paris (AEA 18-051). Mathisson continued to work on partial differential equations and published, in 1934, in a Polish journal, another paper on the subject [M5].

In 1935, at the invitation of Jacques Hadamard and Paul Langevin, Mathisson went to Paris and gave lectures at the Collège de France on differential equations of the hyperbolic type and the diffusion of waves in Riemannian spaces (HU).

1936–37: appointment in Kazan

On 3 November 1935 Einstein wrote to Hadamard saying that Mathisson could come for a year to the Institute for Advanced Study in Princeton and asking for his current address (AEA 18-053). He also wrote a letter to Mathisson, sent to Paris and containing an invitation for Mathisson (no copy

in the Archive). The letters were forwarded by Mlle Jacqueline Hadamard, a daughter of the mathematician, to Mathisson who was then in Russia (as were many scientists during the 1930's: Jakob Grommer, Nathan Rosen, Victor Weisskopf, and others).

Presumably, the letters took a long time to reach Mathisson; his reply was dated "Moscou, le 23 Juin 1936" (AEA 18-054). Delighted by the invitation, Mathisson wrote that he could come to Princeton for the academic year 1937–38, after a year spent in Moscow and Kazan. He also commented favorably on the conditions of work in Kazan and implied that he would go back to Kazan from Princeton.

From a letter of Prof. B.L. Laptev we know that Mathisson arrived in Kazan on May 3, 1936 (BLL), but from Mathisson's letter it follows that he was spending a part of the summer of that year in Moscow.

We do not know how and by whom Mathisson's appointment in Kazan had been arranged.

In a letter of 7 July 1936, Einstein expressed delight with the news that Mathisson had found good conditions to work in Soviet Russia (AEA 18-055). He wrote that, considering how many scientists had then been deprived of the possibility to work, it would not be right to invite Mathisson to the Institute for Advanced Study. In a postscript, Einstein mentions that he has shown, with Rosen, that there are no gravitational waves. There is an excellent account by Daniel Kennefick [7] of the events that occurred in connection with an attempt by Einstein to publish a paper with Rosen purporting to show the non-existence of plane gravitational waves.

Mathisson's next letter, from Kazan, is dated 18 April 1937 (AEA 18-057); he refers to Einstein's results on gravitational waves as being in agreement with his work on Huygens' principle and the diffusion of waves in curved spaces. He expects to travel in June to Paris and gives, for correspondence, his mother's address in Warsaw (Leszno 47, not as good as the previous one).

A letter of 7 May 1937 (AEA 18-058), is the last, in the Einstein Archives, from Einstein to Mathisson. It is a little cooler than earlier letters; Einstein wrote about his collaboration with "your colleague" Leopold Infeld (and with Banesh Hoffmann). They have developed a new method (later called the EIH method) of deriving the equations of motion of point masses and have shown that there are no additional conditions that could be interpreted as corresponding to quantum phenomena [8]. By May 1937, arrangements had already been made with Leopold Infeld to come to the Institute for Advanced Study in order to work with Einstein. Infeld had asked about the possibility to do so in February 1936, in early May his salary for a one-year stay at the Institute had been granted, and Infeld eventually arrived in Princeton in October 1937 [9].

Mathisson left Kazan at the end of May 1937, and went on a short visit to Paris; in a letter sent to Einstein on 5 September 1937 (AEA 18-061), he explained that he would not return to Kazan because “already at the end of May the situation of a foreigner there was unbearable”; he left behind all his belongings and books (presumably so as not to let the Soviet authorities know of his intentions). Recall that in 1937 Stalin’s purges and spy-hunting were intensifying.

A few months after returning from Kazan, Mathisson sent to *Acta Physica Polonica* his most important paper [M6]. In this paper, Mathisson introduced the notion of a ‘gravitational skeleton’ and gave a derivation of the coupling between spin and curvature. In Mathisson’s definition of the gravitational skeleton one can see the germ of the idea of a distribution, as later introduced by Laurent Schwartz: Mathisson uses “test functions” $p_{\mu\nu}$ and uses the equation

$$\int_D T^{\mu\nu} p_{\mu\nu} d^4x = \int_L (m^{\mu\nu} p_{\mu\nu} + m^{\mu\nu\rho} p_{\mu\nu;\rho} + \dots) ds, \quad (1)$$

to replace the continuous energy-momentum tensor $T^{\mu\nu}$ filling a world-tube D by an equivalent — as far as the external field is concerned — distribution with support on a time-like world-line $L \subset D$. Taking $p_{\mu\nu} = \xi_{\mu;\nu} + \xi_{\nu;\mu}$ and using the conservation law $T^{\mu\nu}{}_{;\nu} = 0$, Mathisson proves that the above integral over L vanishes for arbitrary ξ vanishing at both ends of the world-line (“Mathisson’s variational principle”). He then uses this principle to derive the equations of motion, which in modern form are [10]

$$\dot{p}_\mu + \frac{1}{2} R_{\mu\nu\rho\sigma} u^\nu s^{\rho\sigma} = 0, \quad (2)$$

$$\dot{s}_{\mu\nu} + u_\mu p_\nu - u_\nu p_\mu = 0, \quad (3)$$

where p^μ and $s^{\rho\sigma}$ are, respectively, the momentum and spin (intrinsic angular momentum) of a body moving in spacetime with curvature described by the Riemann tensor $R_{\mu\nu\rho\sigma}$; dots denote covariant derivatives in the direction of u . They were derived also by Achilles Papapetrou [11] and are now often called the *Mathisson–Papapetrou equations*. In the rest frame of the body, spin is presumed to have only space components; this is expressed by the Frenkel condition [12]

$$s_{\rho\sigma} u^\sigma = 0, \quad (4)$$

which leads to $p_\mu = m u_\mu + s_{\mu\nu} \dot{u}^\nu$, where $m = p_\mu u^\mu$. Equations (2) and (3) subject to (4), in the limit of special relativity, have “helical” solutions that Mathisson interpreted as being classical counterparts of the quantum *Zitterbewegung* [M7]. Tulczyjew [13] proposed to replace (4) by the condition

$s_{\rho\sigma}p^\sigma = 0$ which, in special relativity, together with (2) and (3), implies that the center-of-mass line L is straight, $p_\mu = mu_\mu$ and $\dot{s}_{\mu\nu} = 0$. The equations (2) and (3) have been the subject of much research and were thoroughly discussed at the conference; they were also derived for a Weyssenhoff–Raabe fluid with spin, within the framework of the Einstein–Cartan theory [14].

In September 1937 Mathisson attended a conference at the Institute for Theoretical Physics (now Niels Bohr Institute) in Copenhagen (NBA).



Fig. 4. Mathisson, Wolfgang Pauli and Gerhard Heinrich Dieke in Copenhagen (1937). Photo: AIP.

Candidacy for the chair for theoretical physics in Jerusalem

For some time, Mathisson was being considered for a chair for theoretical physics at the Hebrew University of Jerusalem. The search for this position went on for a number of years, and it was difficult to find a suitable candidate for the task [15]. Mathisson's name seems to appear for the first time in a letter by Adolf Abraham Ha'levi Fraenkel, then dean of the faculty of sciences of the Hebrew University, to Einstein of 18 June 1936. After Eugene Wigner's candidacy had fallen through, Fraenkel mentioned Mathisson as a candidate together with Infeld and also Louis Goldstein of Paris. In his response, Einstein proposed Lothar Nordheim and Cornelius L  nczos over Mathisson and Goldstein.

In 1937, Einstein's opinions about Mathisson with respect to the position in Jerusalem were curiously intertwined with their mutual work on the problem of motion. On 18 April 1937, Mathisson wrote to Einstein: "I am happy to tell you that I have solved the quantum problem as a dynamical problem of the general theory of relativity..." (AEA 18-057). Ten days later,

Einstein recommended Infeld over Mathisson for the position in Jerusalem in a letter to Salman (Shlomo) Schocken, member of the governing board of the university (AEA 37-251). A week later, on 7 May 1937, Einstein responded to Mathisson's earlier letter, saying: "your indications remain completely incomprehensible to me..." (AEA 18-058). Several weeks later, on 16 June 1937, the EIH paper [8] was submitted to *Annals of Mathematics*. On 4 July 1937, Mathisson wrote to Einstein again announcing 'exact results' and asking him to forward these to *Physical Review*. Just a few days earlier, on 1 July 1937, Fraenkel had written to Einstein that they are considering Infeld "even though we have received new recommendations about Mathisson" (AEA 37-254). Fraenkel probably referred to a letter, written in Hebrew, dated "Warsaw, 13 June 1937", and signed א (aleph), which highly praised Mathisson for the position in Jerusalem (HU). In response to Fraenkel's letter, Einstein wrote on 22 July 1937 to Fraenkel, confirming his assessment that Infeld would be more suitable for the position than Mathisson who seemed to him a little "crazy" (AEA 37-255). It is just a few weeks later, on 5 September 1937, that Mathisson submitted his own most important paper on the problem of motion to the *Acta Physica Polonica*.

Einstein defended his assessment of Mathisson over Infeld in correspondence with Hadamard in April 1938 (AEA 12-042, 12-043). In August 1938, he preferred Fritz London over Mathisson, whom he could only recommend if someone else would be available to lecture about quantum theory (HU). But later in 1938, he recommended Mathisson again for the position, when the alternative candidate was Reinhold Fürth (AEA 37-271, 37-273). As late as August 1939, Mathisson wrote to Tullio Levi-Civita from Cambridge that he had lost his position in Poland, that Hadamard was supporting his candidacy for the chair in Jerusalem, and that he was hoping Levi-Civita could recommend him to Fraenkel [34, p. 185]. By that time, however, a decision about the position in Jerusalem had almost been reached. It was filled in the fall of 1939 by Giulio Racah who succeeded in the following decades to build up a strong center for theoretical physics in Jerusalem [15].

1938–39: Cracow

Some time at the end of 1937 or the beginning of 1938, Mathisson went to Cracow at the invitation of Jan Weyssenhoff, who, in 1935, had been appointed as professor of theoretical physics at the Jagellonian University. Unlike in Warsaw, he found there a congenial atmosphere to work; he collaborated with Jan Weyssenhoff, Józef K. Lubański and Adam Bielecki. Some time in the 1960s, Weyssenhoff told his then Ph.D. student Andrzej Białas that it was Mathisson who had explained to Lubański how to construct, from the spin bivector $s^{\mu\nu}$, the object that is now known as the *Pauli–Lubański vector*.

Weyssenhoff found financial support from private sources for Mathisson. According to Andrzej Schinzel, Leon Rappaport, Mathisson's colleague from Warsaw University, was among those helping him. Leon Rappaport wrote later a book [16]; its first chapter is devoted to Mathisson who appears under the cryptonym *Radon*.

A young mathematician from Cracow, Irena Jungermann, became Mathisson's wife.

His stay in Cracow exerted a long-lasting influence on research in theoretical physics there; Weyssenhoff and his students continued to work on the motion of particles with structure in gravitational and electromagnetic fields until the late 1960s. Especially important was the extension, due to Weyssenhoff and Antoni Raabe [17], of Mathisson's ideas to continuous media and, in particular, the development of a relativistic theory of 'spinning fluids'. By integration, they obtained, from the equations for a dust with spin, the Mathisson equations for individual particles. Continuing the ideas of [M7], they found solutions of those equations corresponding to the motion of an electron on a circle. In another paper, they considered the motion of spinning particles with the speed of light [18]. Weyssenhoff compared the properties of a classical particle with spin with those described by the Dirac equation [19]. A review of that work is given in Ref. [20, 21]; it is mentioned in several monographs [22–24]. A recent moving homage to Mathisson, Weyssenhoff, Raabe and the influence of their work is in the article by Peter Horváthy [25].

1939–40: Paris and Cambridge

In the spring of 1939, the Mathissons went to Paris. Presumably during the stay there, Mathisson wrote a short note on *Le problème de M. Hadamard relatif à la diffusion des ondes*, that Hadamard presented to the *Comptes Rendus* [M8]. Later that year, a longer paper under the same title appeared in the Swedish *Acta Mathematica* [M9]. It is considered to be the most important mathematical paper by Mathisson: it presents the first proof, in a special case, of Hadamard's conjecture on the class of hyperbolic differential equations that satisfy Huygens' principle.

Later that year, the Mathissons went to Cambridge, where Myron continued to work on his method of deriving equations of motion. In a paper [M11] communicated for him by P.A.M. Dirac on 9 February 1940, he gives a simplified derivation of his fundamental formula (1) and of the resulting variational principle. It is worth noting that even though Warsaw University never offered him a job, he was loyal to it and, even in 1940, when he could not envisage ever returning there, he indicated that university as his affiliation. Some time in 1940, the news reached him that Czesław Białobrzęski was killed in Warsaw by the Nazis. Mathisson wrote his obituary [26]. The

tragedy concerned a different person of the same name; Professor Czesław Białobrzeski lived in Warsaw until his death in 1953.

Suffering from tuberculosis, Myron Mathisson died in England on 13 September 1940.

After the war, his widow, then Mrs Gill, settled in Rhodesia. In the 1970s, Bruno Lang, a professor of radio chemistry in Warsaw, Peter Havas and Stan Bażański exchanged a few letters with her. She mentioned having seen a letter from Einstein; according to her, he wrote “you must be my natural son, and we must meet and talk about this”. From those letters we know that Myron’s parents came to Warsaw from Riga, that Myron was not an easy man to reach and to get to know, and that he died of tuberculosis, not of hunger, as suggested by Infeld in his autobiographical sketch *Kordian, fizyka i ja*.

Mathisson had good contacts with physicists in Cambridge (DA); in the first paper written there he thanks M.H.L. Pryce for valuable suggestions. Mathisson made an impression on P.A.M. Dirac, who edited and published, posthumously, his last paper [M12] and wrote his obituary for *Nature* [27].

Hadamard was so impressed by the work of Mathisson that, after his death, living then in New York as a refugee from Nazi-occupied Paris, he published in the prestigious *Annals of Mathematics* a paper dedicated to Mathisson and containing an exposition of his result [28]. It begins with the words “To the memory of Myron Mathisson, whose premature death is a cruel loss to Science, I dedicate this treatment of the problem which he has solved so beautifully”.

Scientific publications of Myron Mathisson

1. Die Beharrungsgesetze in der allgemeinen Relativitätstheorie, *Z. Phys.* **67** (1931) 270–277.
2. Die Mechanik des Materieteilchens in der allgemeinen Relativitätstheorie, *Z. Phys.* **67** (1931) 826–844.
3. Bewegungsproblem der Feldphysik und Elektronenkonstanten, *Z. Phys.* **69** (1931) 389–408.
4. Eine neue Lösungsmethode für Differentialgleichungen von normalem hyperbolischem Typus, *Math. Annalen* **107** (1933) 400–419 and 648 (Berichtigung).
5. Die Parametrixmethode in Anwendungen auf hyperbolische Gleichungssysteme, *Prace Matematyczno-Fizyczne, Warszawa* **41** (1934) 177–185.
6. Neue Mechanik materieller Systeme, *Acta Physica Pol.* **6** (1937) 163–200. An English translation by Anita Ehlers is due to appear as a ‘Golden Oldie’ in *Gen. Rel. Grav.*, forthcoming.

7. Das zitternde Elektron und seine Dynamik, *Acta Phys. Pol.* **6** (1937) 218–227.
8. Le problème de M. Hadamard relatif à la diffusion des ondes, *Comptes Rendus de l'Académie des Sciences, Paris* **208** (1939) 1776–1778.
9. Le problème de M. Hadamard relatif à la diffusion des ondes, *Acta Math.* **71** (1939) 249–282.
10. Sur un théorème concernant une transformation d'intégrales quadruples en intégrales curvilignes dans l'espace de Riemann, *Bull. Internat. Acad. Polonaise Sci. Lett., Cracovie Cl. Sci. Math. Natur., Sér. A* (1939) 22–28 (co-authors: A. Bielecki and Jan W. Weyssenhoff).
11. The variational equation of relativistic dynamics, *Proc. Cambridge Phil. Soc.* **36** (1940) 331–350.
12. Relativistic dynamics of a spinning magnetic particle, *Proc. Cambridge Phil. Soc.* **38** (1942) 40–60 (published posthumously).

Appendix: the sources

Our account of Mathisson's life is based on the following sources: Most important are several articles [20], [29] and [30] by Bronisław Średniawa, now an Emeritus Professor at the Jagellonian University. In 1938, as a student at that university, Średniawa met Mathisson in person. Later, he prepared a Ph.D. thesis, supervised by Jan Weyssenhoff and devoted to an application of the Mathisson variational principle to the problem of motion of dipole and quadrupole particles [31]. During the last 25 years Średniawa was active in describing and presenting at conferences the history of work on relativity in Cracow. He contributed, more than anyone else, to keeping alive the memory of Mathisson and his collaboration with Weyssenhoff.

Another important source is the article by Peter Havas [32]. In this historical account, Havas is very critical of the work on the problem of motion done by Einstein and (especially) Infeld; he gives much credit to Mathisson and writes that “Mathisson's contributions were far more original than Infeld's and introduced far better mathematical methods” [32, p. 267]. He speculates that “if Einstein had succeeded in getting Mathisson to join him in Berlin or Princeton ... there would have been no EIH, but presumably an EMH ...” [32, p. 268]. Infeld himself also mentions Mathisson in his autobiography [33, 203–205].

Among the unpublished sources, the most important one is the Einstein–Mathisson correspondence as well as related documents in the Albert Einstein Archives (AEA) at the Hebrew University of Jerusalem, see note 2. Unpublished documents from the Albert Einstein Archives are quoted by kind permission. Further documents, including a CV dated 1937, are contained

in the Hebrew University's archives (HU), folder 'physics 1938'; we wish to thank Issachar Unna for his assistance in locating these documents. We are also grateful to Stanisław Bażański who made available to us his excerpts from the University of Warsaw Archive; they are referred here to as (UW). He also supplied us with two letters from Mrs Irena Gill (IG), Mathisson's widow, the portrait of Mathisson reproduced as Fig. 1, and a letter, dated 30 October 1978, from Prof. B.L. Laptev, University of Kazan (BLL). We also thank Włodzimierz Zych who helped us to obtain information about Mathisson's studies at Politechnika Warszawska (PW).

The Rockefeller Archive Center in New York (RA) contains documents pertaining to Einstein's visit and intervention about Mathisson (RFam, RG2, Ser. H, Box 162, Folder 1256). The Paul Dirac papers at Florida State University in Tallahassee (DA) contain two letters by Mathisson and one by his widow, dated 5 February, 7 April, 22 November 1940, respectively, as well as a letter by Hadamard to Dirac, dated 14 November 1940. The Niels Bohr Archives (NBA) in Copenhagen contain a letter by Mathisson, dated 7 August 1937, as well as a response by Bohr (Suppl. Sci. Corr.). Finally, three letters by Mathisson to Levi-Civita, dated 4 November 1932, 26 September 1937, and 25 August 1939, as well as two letters by Hadamard to Mathisson, dated 4 November 1931 and 9 February 1932, were published in [34]. We also thank Diana Buchwald, Margaret Hogan, Rosy Meiron, Felicity Pors, Sharon Schwerzel, and Rossana Tazzioli for further information and assistance.

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